Hydrodynamics of mountain rivers and streams

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Presentation schedule

1. Introduction – basis of fluid movement
2. Exemplary velocity profiles and shear stresses calc.
3. Pictures
4. Lowland and mountain rivers - comparison
5. Bedload characteristics
6. Hydrodynamical balance of river channel
introduction
fluid movement in open channels – real flow – Bernoulli equation

\[ h_1 + D_1 + E_{vel1} = h_2 + D_2 + E_{vel2} + E_{loss} \]

\[ \frac{\alpha V^2}{2g} \]

\[ D_1 \]

\[ h_1 \]

datum line

\[ \frac{\alpha V^2}{2g} \]

\[ E_{loss} \]

\[ D_2 \]

\[ h_2 \]
Within the cross-section I and II the velocity distribution differs in real flow conditions $\alpha_1 \neq \alpha_2$

$\alpha \in <1, 2>$ (in channels the coefficient vary around the 1.15)

according to flow: 1 – when turbulent, 2 - when laminar
introduction
fluid movement in open channels – specific energy

specific energy = potential + kinetic

\[ \frac{\alpha V_2^2}{2g} \]
introduction
nature of fluid movement in open channels – specific energy

sub-critical flow – low velocity, high depth, potential energy predominate
super-critical flow – high velocity, low depth, kinetic energy predominate
introduction
nature of fluid movement in open channels – Froude Number

\[ Fr = \frac{U}{\sqrt{gD}} \]

also used as:

\[ Fr = \frac{U^2}{gD} \]

where:

- \( U \) – water velocity,
- \( D \) – hydraulic mean depth (cross sectional area of flow / top width)
- \( g \) = gravity

\( Fr = 1 \), critical flow,
\( Fr > 1 \), supercritical flow (fast, rapid flow),
\( Fr < 1 \), subcritical flow (slow / tranquil flow)

sub-critical flow – low velocity, high depth, potential energy predominate
super-critical flow – high velocity, low depth, kinetic energy predominate
introduction
nature of fluid movement in open channels – Froude Number

\[
Fr = \frac{U}{c}
\]

where:

- \( U \) – characteristic velocity,
- \( c \) – characteristic water wave propagation velocity

The Froude number is thus analogous to the Mach number.
Types of flow

time criterion:
steady – unsteady

depth change or doesn't change with time

Q = V A – assumption

space as criterion:
uniform – nonuniform (varied)

Thus flow can be:
1. steady and uniform (fundamental for most calculations),
2. steady and nonuniform (e.g. upstream and downstream to the tributary),
3. unsteady and uniform (e.g. waves in the channel),
4. unsteady and nonuniform (e.g. during inundation).
dynamic (shear) viscosity
\[ \mu \left[ \frac{Ns}{m^2} \right] \]

\[ F = \mu A \frac{u}{y} \left[ N \right] \]

\[ \tau = \mu \frac{\delta u}{\delta y} \left[ \frac{N}{m^2} \right] \]

kinematic viscosity
\[ \nu = \frac{\mu}{\rho} \left[ \frac{m^2}{s} \right] \]

\[ \tau_0 = \gamma DI \]

shear velocity
\[ U_\ast = \sqrt{\frac{\tau_0}{\rho}} \left[ \frac{m}{s} \right] \]
Laminar flow appears only in very slow motion or in near bed viscous sublayer. Due to turbulence water layers are mixed.
In pipes it is assumed that laminar flow appears for $Re < 2300$ and turbulent flow appears for $Re > 4000$. In rivers $Re$ mostly exceeds 100 000.

Reynolds number $Re = \frac{UY}{v}$

source: wikipedia

Yalin 1977
Reynolds number \[ Re = \frac{UY}{v} \]

For flow conditions in open channels the Grain Reynolds Number is also being used

\[ Re^* = \frac{U^* d_s}{v} \]

Flow conditions:

smooth: \[ 0 < Re < 3.63 - 5 \]

transitional: \[ 3.63 - 5 < Re < 68 - 70 \]

rough: \[ Re > 68 - 70 \]

Sentürk 1977
introduction – nature of fluid movement in open channels – viscous forces

where:
L - mixing length
Y – flow depth

within the open channel flow the two borders can be indicated:
- bottom,
- water surface.
introduction – nature of fluid movement in open channels – zero plane

rough bed – gravel, boulders, stones

smooth bed – sand
introduction – nature of fluid movement in open channels – velocity profile

Prandtl von Karman equation

\[ \frac{U}{U_*} = \frac{1}{\kappa} \ln \frac{d U_*}{v} + B \]

\[ \kappa = \frac{1}{\sqrt{2\pi}} \sim 0.4 \]

Flow conditions:
smooth: \( 0 < \text{Re} < 3.63 - 5 \)
transitional: \( 3.63 - 5 < \text{Re} < 68 - 70 \)
rough: \( \text{Re} > 68 - 70 \)
introduction – nature of fluid movement in open channels – velocity profile

Prandtl von Karman equation

\[
\frac{U}{U_\ast} = \frac{1}{\kappa} \ln \frac{d U_\ast}{v} + B
\]

Karman constant

\[\kappa = \frac{1}{\sqrt{2\pi}} \sim 0.4\]

Flow conditions:

hydraulically smooth

\[
\frac{U}{U_\ast} = \frac{1}{\kappa} \ln \left( \frac{y U_\ast}{v} \right) + 5.5
\]

hydraulically rough

\[
\frac{U}{U_\ast} = \frac{1}{\kappa} \ln \left( \frac{y}{k_s} \right) + 8.5
\]

or

\[
\frac{U}{U_\ast} = 5.75 \log \left( \frac{y}{k_s} \right) + 8.5
\]

\[
\frac{U}{U_\ast} = 5.75 \log \left( \frac{30 y}{k_s} \right)
\]
velocity profiles – grain size – monofraction 4 cm

wake region

\( \frac{y}{Y} \) [ - ]

\( \frac{U}{U_M} \) [ - ]

Liczba Froude'a [ - ]

- 1.624
- 1.562
- 1.382
- 1.127
- 0.815
- 0.662
- 0.481
- 1.020
- 1.597
- 1.411
- 1.238
- 0.872
- 0.675
- 0.384
- 0.074
- 0.254
velocity profiles – grain size – 4 cm

Prandtl von Karman equation

\[ \frac{U}{U_*} = \frac{1}{\kappa} \ln \left( \frac{d U_*}{\nu} \right) + B \]

bottom

surface
velocity profiles – various grain sizes

Slope of the profiles generalized within the same bed roughness vary
velocity profiles – various grain sizes

\[ U_* = \sqrt{\frac{\tau_0}{\rho}} \quad [\text{m/s}] \]

\[ \tau_0 = \gamma DI \]

\[ \frac{U}{U_*} = \frac{1}{\kappa} \ln \frac{d U_*}{v} + B \]
measured velocity profiles – shear stresses calculation

\[ \frac{U}{U_*} = \frac{1}{\kappa} \ln \frac{y U_*}{\nu} + B \]

\[ U_* = \frac{s}{5.75} \]

\[ \tau_0 = \rho U_*^2 \]

Gordon and McMahon 1992
Movies visualizing these phenomena

Reynolds number
http://www.youtube.com/watch?v=eIHVh3cIujU
http://www.youtube.com/watch?v=NplrDarMDF8
http://www.youtube.com/watch?v=kmjFdBxbV08

Froude number
http://www.youtube.com/watch?v=Q0R5xr-BqdA
http://www.youtube.com/watch?v=cRnIsqSTX7Q
Lowland and mountain rivers - comparison

[Bradshaw i inn. 1978, Schumm 1977]
Lowland and mountain rivers - comparison

- Flood in mountain rivers – hours
- Flood in lowland rivers – days, weeks
Lowland and mountain rivers - comparison

relative roughness = \frac{\text{grain or bedform size}}{\text{flow depth}}
Lowland and mountain rivers - comparison

Radecki-Pawlik 2011

lowland rivers
Lowland and mountain rivers - comparison

![Graph showing comparison between lowland and mountain rivers. The graph plots dimensionless shear stress against boundary Reynolds number. The x-axis represents the boundary Reynolds number, $R_e = \frac{U_d d_s}{v}$, and the y-axis represents the dimensionless shear stress, $\tau^* = \left( \frac{\tau}{\rho g d_s^2} \right)$. The graph includes a Shields curve and a fully developed turbulent velocity profile. Various materials are tested, such as Amber, Lignite (Shields), Granite, Barite, Sand (Casey), Sand (Kramer), Sand (U.S. WES.), Sand (Gilbert), Sand (White), Sand in air (White), and Steel shot (White).]
Lowland and mountain rivers - comparison

Hjulstrøm 1939
bed stability – incipient motion

<table>
<thead>
<tr>
<th>Shape</th>
<th>Incipient Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>$\psi_p = 0.75$</td>
</tr>
<tr>
<td>Oblate Ellipsoid</td>
<td>$\psi_p = 0.60 \div 0.86$</td>
</tr>
<tr>
<td>Elongated Ellipsoid</td>
<td>$0.50 \leq \psi_p \leq 0.75$</td>
</tr>
<tr>
<td>Disk</td>
<td>$0.65 \leq \psi_p \leq 0.74$</td>
</tr>
<tr>
<td>Elongated Disk</td>
<td>$\psi_p = 0.60$</td>
</tr>
<tr>
<td>Cube</td>
<td>$\psi_p = 0.75$</td>
</tr>
<tr>
<td>Prism</td>
<td>$0.6 \leq \psi_p \leq 0.75$</td>
</tr>
<tr>
<td>Board</td>
<td>$\psi_p \leq 0.74$</td>
</tr>
<tr>
<td>Board</td>
<td>$\psi_p = 0.6$</td>
</tr>
<tr>
<td>Rod</td>
<td>$\psi_p = 0.5$</td>
</tr>
</tbody>
</table>
Bed stability – incipient motion

\[ \tau_{cr 	ext{ i}} = f_i \Delta \gamma_s d_i \]

where:

- \( f_i \) – critical Shields stresses for diameter “i”
- \( \Delta \gamma_s \) – specific weight of submerged sediment
- \( d_i \) – characteristic diameter

Drag force:

\[ F_x = C_x \frac{1}{2} A \rho_0 U^2 \]

Lift force:

\[ F_y = C_y \frac{1}{2} A \rho_0 U^2 \]
bed stability – incipient motion

mountain river bed granulometry

Skawa - sieve curve

<table>
<thead>
<tr>
<th>Parameter (d)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>d16 [m]</td>
<td>0.021</td>
</tr>
<tr>
<td>d50 [m]</td>
<td>0.072</td>
</tr>
<tr>
<td>d84 [m]</td>
<td>0.133</td>
</tr>
<tr>
<td>d90 [m]</td>
<td>0.137</td>
</tr>
<tr>
<td>dm [m]</td>
<td>0.077</td>
</tr>
<tr>
<td>δ [-]</td>
<td>2.55</td>
</tr>
<tr>
<td>SF [-]</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Hydrodynamical balance of river channel

\[ \tau_{cr} = f_m \Delta \gamma_s d_m \]

\[ \tau_0 = \gamma DI \]

\[ \tau_0 = \tau_{cr} \]

\[ \tau_{cr} = \frac{f_m \Delta \gamma_s d_m}{\gamma DI} \]

---

<table>
<thead>
<tr>
<th>h (Q %) [m]</th>
<th>initial</th>
<th>armored Q5 - 1.88</th>
<th>Q50%</th>
<th>Q1%</th>
<th>Qb</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta ) [-]</td>
<td>1.49</td>
<td>1.10</td>
<td>1.53</td>
<td>1.53</td>
<td>1.30</td>
</tr>
</tbody>
</table>

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Wolman i Miller 1960
Hydrodynamical balance of river channel

Skawa

1st terrace

a - Q10%
b - Q25%
c - Q50%
d - Q75%

bankfull

Czarny Dunajec

Q75%  Q50%  Q10%

1st terrace

bankfull

flow depth [m]
Hydrodynamical balance of river channel

hydrodynamical balance

flow capacity ≈ load (bed, suspended) capacity
Hydrodynamics of mountain rivers and streams

Thank You