Abstract: Calculation of shear stresses in rapidly flowing water over rough bed can be supported by measurements of velocity profile and bed roughness. New method of bed roughness measure is proposed to compare the roughness of Nikuradse. Froude and Reynolds numbers in mountain rivers flow reach high volumes which causes velocity profiles differ from the ones found in lowland rivers. Four zones of flow can be found in velocity profile but for hydraulically rough conditions, the free surface influence zone takes an average 20% of flow depth and laminar flow zone becomes very thin. Calculation of shear stresses over rough bed is accurate if logarithmic equation is used.

KEY WORDS: velocity profile, shear stresses, bed roughness, log-law, flow zones

1. LABORATORY MEASUREMENTS

Small mountain rivers flow conditions were simulated in laboratory flume with glass walls. Bed observations in 62 measurement series with artificial grains were performed. The flume dimension was 12x0.5x0.6 m. Bed slopes varied from –8.4 to 50.9 ‰ and flow depth was between 0.05 and 0.47 meters. Temporary velocity in flow direction was measured using micro-propeller connected to PC. Each series lasted 20-45 minutes and had comprised 2 minute measurements made every 2 cm step vertically.

Grains used in laboratory measurements

<table>
<thead>
<tr>
<th>Symbol</th>
<th>H [cm]</th>
<th>Ø [-]</th>
<th>SF [-]</th>
<th>Z [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4M</td>
<td>1.50</td>
<td>3.80</td>
<td>0.89</td>
<td>60.0</td>
</tr>
<tr>
<td>4D</td>
<td>2.30</td>
<td>3.90</td>
<td>0.92</td>
<td>62.6</td>
</tr>
<tr>
<td>6M</td>
<td>1.45</td>
<td>4.70</td>
<td>0.79</td>
<td>52.4</td>
</tr>
<tr>
<td>6D</td>
<td>3.15</td>
<td>5.50</td>
<td>0.93</td>
<td>68.1</td>
</tr>
<tr>
<td>8M</td>
<td>1.80</td>
<td>6.60</td>
<td>0.74</td>
<td>46.8</td>
</tr>
<tr>
<td>8D</td>
<td>3.65</td>
<td>7.80</td>
<td>0.97</td>
<td>61.5</td>
</tr>
<tr>
<td>8A</td>
<td>3.65</td>
<td>7.80</td>
<td>0.97</td>
<td>42.9</td>
</tr>
</tbody>
</table>

H – grain height, Ø – grain diameter, SF – shape factor, Z – percent of bed covered by grains.
The number of measurements and duration of gathered velocity data depended on water depth and velocity of flowing water. This allowed precise plotting of velocity profiles. Additionally energy slope and bed roughness were measured. Fixed grain sizes were 4, 6 and 8 centimeters (tab. 1). The shape range was $SF = <0.74-0.97>$ ($SF = a/\sqrt{bc}$). Used grain shape was hemisphere ($H = R$) and hemisphere cut off ($H = \frac{1}{2}R$) substituting ellipsoidal grains often found in Carpathian Rivers. The identical roughness simulated the armor coat with fine grains removed from the bed surface which permitted uniform flow conditions. Grains were packed on bed covering from 42.9 to 68.1 percent of its area (table 1, figure 1).

Bed roughness was described by the profile-meter AG-1 [Bartnik 1992] (fig. 2). The method does not consider sorting and hiding processes. Measurement describes the bed resistance to flowing water. This allowed the use of profile-meter to describe uniform and non-uniform bed load as well as for calculating velocity with Prandt’l von Karman’s equation.

Profile-meter telescope plummet lengths after impress on the grains are proposed to calculate standard deviation ($K$) (eq. 1) [Bartnik et al 2001].

$$K = \sqrt{\frac{1}{n-1} \sum_{n=1}^{N} (h_n - H)^2}$$

where: $N$ – amount of profile-meter telescope plummets. $h_n$ – $n^{th}$ plummet length. $H$ – mean profile-meter distance from bed, $K$ – bed roughness measure.
$K$ parameter cannot be directly used in log-law function. Sand roughness $k_s$, found in Prandtl von Karman’s equation for uniform bed roughness can be obtained using profile-meter. Roughness measure $K$ together with shape factor $SF$ allow defining sand roughness $k_s$ (eq. 2).

$$k_s = K (1.926 \, SF^2 - 0.488 \, SF + 4.516) \quad (2)$$

### 2. THE INFLUENCE OF RELATIVE ROUGHNESS ON VELOCITY PROFILE

#### 2.1. VELOCITY PROFILE MEASUREMENTS AND DISCUSSION

Velocity profile consists of separate 4 flow zones [Williams 1996]. In rough flow conditions ($Re > 70$) laminar layer becomes very thin not surpassing bed surface and wake region height decreases. Measurements done in laboratory flume show that for rough flows condition inverse velocity region reach an average 20% of flow depth. In shallow water flow while Froude number exceeds 1, wake region disappears. Improving accuracy of velocity profile, calculation can be done using Coles “wake function” [Xinyu, Zengan, Changzhi 1995]. The velocity calculation assuming logarithmic arrangement in whole profile gives the results in figure 3. Near bed error for velocity calculated for $Fr=0.074$ ($U_*=0.016$) equals 5.7% decreasing up to 0.53 of relative distances from bed. When approaching water surface calculation velocity error increases to 35%. For subcritical flow conditions ($Fr=1.38, \, U_*=0.195$) mean error reaches 3.6% not exceeding 6% on the water surface.

![Fig. 3. Examplary calculation of velocity using logarithmic equation for grains 4D.](image)

$\square$ – measured, $\Delta$ – calculated

These example plots (fig. 3) show the tendency of flow conditions to influence the shape of velocity profile. Also the relative roughness changes velocity distribution. Grains resting on bed change the maximum velocity level, bed velocity and the slope of
velocity profile. This is presented in figure 4 where the velocity and depth are shown as relative entities ($y/Y$ and $U/U_{max}$). To express the impact of bed the data gathered for measurements over flat bed versus covered by 8D grains (the biggest used) was chosen.

![Figure 4. Velocity profile distribution over: a.) rough b.) flat bed.](image)

For the same roughness plots of velocity profiles are parallel but vertically shifted depending on flow depth. In this case the relative roughness influences the height of maximum velocity appearance. It ranged for presented plots in $y/Y \in (0.68-0.88)$ when 8D grains were used and in $y/Y \in (0.35-0.62)$ for the flow over flat bed. Waving effect influences the accuracy of surface velocity for rapid flows measurements what caused that maximum velocity level might not be properly described and lay higher then measured. Near bed, ($y/Y=0.1$) velocity for flow over flat bed had an average 88% of maximum velocity. The energy used by 8D grains caused that the relative velocity reached an average 30% of the maximum on the same level. The laminar flow zone was hidden in the bed surface layer.
2.2. USE OF LOGARITHMIC EQUATION FOR CALCULATION OF VELOCITY DISTRIBUTION

The run of near bed flow can be treated as logarithmic and used for calculating velocity and shear stresses. One of the problems when using the logarithmic equation is to describe zero velocity level \( y_0 \). While \( A \) value is connected with Karman’s constant, \( B \) value describes flow roughness. Nezu and Rodi [Cardoso et al. 1989] state that the change of constant \( B \) in Prandtl von Karman’s equation reaches 38% for hydraulically smooth condition flow. For rough conditions \( B \) value is often treated as a 8.5 which allows modification of standard equation for sand roughness \( k_s \). Bed level (zero velocity) is calculated as \( 1/30 \) of \( k_s \).

\[
U = 5,75 U_* \log \left( \frac{30 y}{k_s} \right) . \tag{3}
\]

When the shape of grains varies, sorting and hiding effect appear, \( k_s \) value varies for the same bed load. Velocity profile cannot be plotted properly and velocity gradient near bed calculation fails. To skip the sand roughness and shear velocity measurements, the \( U/U_* \) can be replaced by \( U/U_{max} \) and \( y/k_s \) by \( y/Y \) [Strużyński 2001]. This substitutes shear velocity with the maximum velocity calculation and limits using the method only to rough flow conditions where the highest velocity appears near water surface. However, the accuracy of velocity and velocity gradient near surface calculation increase, comparing to standard equation, with the properly described flow depth.

\[
\frac{U}{U_{max}} = A \log \left( \frac{y}{Y} \right) + B \tag{4}
\]

Replacing of sand roughness causes that bed elevation does not influence the change of constant \( B \) that becomes equal to \( 1.12 \pm 3\% \). The plot of data gathered in laboratory flume shown in figure 5 describes the high correlation between constant \( A \) and relative roughness \( Y/K \).

![Fig. 5. An estimated curve of constant A vs. relative roughness Y/K.](image)
Using profile-meter AG-1 allowed calculating slope of velocity profile and its direct use in equation 4 (eq. 5).

\[ A = 6.38 \left( \frac{Y}{K} \right)^{-0.835} \]  

(5)

Dependence of \( A \) value was verified with correlation factor \( r^2 = 0.72 \) for 69 measurement series with artificial grains (Fig.6). The verification was done for input of average flow velocity.

![Fig. 6. Verification of calculation A constant on relative roughness.](image)

Wake region influences the accuracy of results and the error of calculated velocity equals to ± 0.2 [m s\(^{-1}\)] which means that the relative error decreases from 50 % for 0.2 [m s\(^{-1}\)] to 20% for 1.0 [m s\(^{-1}\)].

### 3. SHEAR STRESSES CALCULATION

Radecki [1992] following Gordon et al [1992] proposes calculation of shear stresses by shear velocity \( U_* = s/5.75 \). The \( s \) value means a slope of velocity profile in a part of logarithmic distribution. Strużyński [2001] uses equations (4) and (6) for calculating shear stresses basing on an \( A \) value. Presuming that logarithmic distribution of flowing water \( du/\Delta y \) is replaced by \( du/\Delta y \), shear stresses are calculated on bed surface \( (y = K/10) \) (eq. 6).

\[ \frac{du}{dy} = \frac{\tau}{\mu + \varepsilon} \]  

(6)
While $\tau_0$ was calculated from laboratory measurements the $\mu + \varepsilon'$ had been approximated as $2.303 \, KU_M$ with correlation exponent $r^2=0.84$. The shear stresses are described by the formula:

$$\tau_0 = 2.303 \, K \, U_M \frac{\Delta U}{\Delta y} \quad (7)$$

where 2.303 is a log to ln conversion factor.

The method was verified with measured data. Laboratory verification of calculated shear stresses was done on 150 series for artificial and natural grains. The average relative appearance of maximum velocity was $y/Y=0.74$. The roughness measure ($K$) was verified as independent from sorting and armoring process for calculating shear stresses with correlation factor $r^2=0.89$. (fig 7).

![Fig. 7. Shear stresses (\(\tau_0\)) calculation to measure verification.](image)

Equation 7 can also be also used for calculating shear stresses in the unique distance from bed. The accuracy of results depends on measured to calculated velocity profile slope agreement. Calculations can be provided within the log-law zone.

**CONCLUSIONS**

Near bed the velocity and velocity profile slope calculations (in logarithmic scale) are correct within the second and third flow zone. The use of equation (4) makes the bed level (zero velocity) estimation error negligible ($B=1.12$).

The use of mentioned method is limited to the rough flow conditions where the maximum velocity lays close to the water surface (the near surface region decreases to 20% of water depth).
The measurements of surface velocity, water depth and bed roughness can be used for calculation of water velocity profile and bed shear stresses for rough flow conditions.

REFERENCES


